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Radially symmetric coherence between satellite gravity and multibeam bathymetry grids

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Abstract We compute the radially symmetric coherence between multibeam bathymetry and satellite gravity grids in 25 areas distributed around the world. In contrast to previous studies employing one-dimensional analysis of data along profiles, our results cannot be biased by unseen off-track topography. The mean coherence averaged over the 20–160 km waveband, and the shortest wavelength at which coherence is above 0.5, vary with tectonic setting. Seamounts and slow spreading ridges have high (>0.7) mean coherence down to ~20 km wavelength, other spreading ridges and trenches have intermediate (0.5 – 0.7) coherence down to ~20 – 30 km wavelength, and continental shelves have low (<0.5) coherence at all wavelengths. In the areas with highest mean coherence, the shortest wavelength at which coherence is above 0.5 decreases as mean depth decreases. The filter employed in the bathymetric prediction method of Smith and Sandwell (1994) selects the most coherent parts of the bathymetry and gravity spectrum.

Keywords Coherence • Satellite gravity • Multibeam • Bathymetry

Introduction

When gravity-topography cross-spectral analysis is performed on data over land, two-dimensional grids covering map areas are usually available. In the oceans, available data are usually limited to profiles along ship tracks, necessitating a one-dimensional analysis. The results of profile analysis may be biased, however, because the gravity anomaly along the profile includes the effects of topography off to the side of the profile. Thus accurate gravity anomalies with real causes can appear to be uncorrelated with the topography along the profile, biasing coherence estimates toward lower values and giving a pessimistic impression of the signal-to-noise ratio in gravity data. The true correlation of intermediate-wavelength gravity and bathymetry anomalies is the basis for design of spectral projection filters that estimate depth from gravity (Smith and Sandwell 1994).

In the last decade, numerous regional multibeam surveys have been collected throughout the world's oceans having sufficient coverage and extent to enable two-dimensional cross-spectral coherence estimates. The ocean-wide distribution of these areas allows us to evaluate coherence in a variety of tectonic settings, and their spatial extents permit assessment of the waveband of interest in bathymetric estimation from satellite-derived gravity. In particular, we examine the shortest wavelength at which coherence exceeds 0.5, which is a critical component in the design of Smith and Sandwell's (1994) spectral projection filters.

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Data

We compiled 25 multibeam grids that met these requirements: (1) survey coverage exceeding 100 km on each side, (2) surveys having relatively complete map coverage with few gaps between swaths, (3) small (< 1000 m) grid spacing, and (4) sampling a variety of geologic settings in the world's oceans. We downloaded multibeam grids from the National Geophysical Data Center, the University of New Hampshire, Lamont-Doherty Earth Observatory, Geoscience Australia, and the University of Hawaii. The locations of the multibeam grids are shown in Figure 1.

Corresponding satellite-derived gravity data are from Sandwell and Smith (1997; version 18.1). Details

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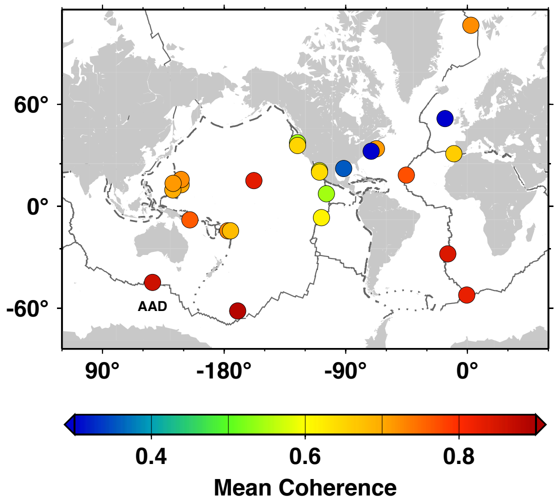

Fig. 1 Twenty-five areas examined in this study. *Colors* indicate mean coherence over the 20–160 km waveband. Tectonic settings are spreading ridges (*solid lines*), trenches (*dashed lines*), continental shelves (offshore California, New England, Gulf of Mexico, Spain, Ireland), and seamounts (Hawaii and offshore New England). Transform plate boundaries (*dotted lines*) had no suitable surveys

for deriving gravity from satellite altimetry are published in Sandwell and Smith (1997, 2009). Their method removes almost all tides and ocean dynamics from altimeter sea surface height measurements and ensures that long (>180 km) wavelengths match the EGM2008 gravity field model. Comparison of satellite-derived gravity anomalies against the most accurate in situ marine gravimetry shows that the Sandwell and Smith product has an accuracy of 2-3 milliGals (Sandwell and Smith 2009).

Coherence Analysis

The cross-spectral coherence between a pair of inputs is the square of the linear correlation coefficient as a function of wavelength, indicating how much of the variance in one input can be correlated with the other input through a linear filtering operation. Coherence near 1 indicates nearly perfect linear correlation, while coherence near 0 generally (von Frese et al. 1997) indicates the absence of any significant linear relationship. A coherence of 0.5 can be interpreted as a signal-to-noise ratio of 1:1 in one input if the other input can be assumed to be noise-free (Bendat and Piersol 1986, Eq. 6.39).

In the context of bathymetric prediction, some part of the gravity anomaly is due to the attraction of seafloor topography, some is due to other (sub-seafloor) sources, and there may be measurement error of a few milliGals. Because modern multibeam measurements are accurate to a few tenths of a percent of depth (Marks and Smith 2009), bathymetry may be considered to be noise-free while gravity has noise; however gravity "noise" in our context is real signal that arises from sub-seafloor sources uncorrelated with seafloor topography. Bathymetric prediction requires the portion of gravity that is correlated with seafloor topography; i.e., the "signal."

Coherence estimation requires averaging of spectral estimates (Bendat and Piersol 1986). We calculate one estimated coherence function for each map area, by azimuthally averaging the spectra. This is done by first obtaining $G(u,v)$ and $B(u,v)$ from Fourier transformations of the gravity $g(x,y)$ and bathymetry $b(x,y)$ grids. Next, the wavenumbers are converted from Cartesian u,v to polar q,θ coordinates. The radial average combines all wavenumbers q falling within a bandwidth $\mathrm{d}q = 1/L$, where L is the length of a side of the (square) grid, and which in our study ranged from about 100 to 450 km. The result is coherence as a function of wavelength, where wavelength is in any and all directions, weighted equally:

$$\text{Coherence} = |\langle GB^* \rangle|^2 / (\langle GG^* \rangle \langle BB^* \rangle)$$

where * is the complex conjugate and brackets < > represent averaging over all θ. This approach is justified because the gravitational field of a point mass is radially symmetric. Therefore, if the topography of the seafloor is of uniform density, the associated gravity anomaly field is related to the topography through a radially symmetric operator.

The satellite-derived gravity grid is on a Mercator projection with 1-minute grid spacing, while the 25 multibeam grids came in a variety of projections and grid spacings, with surveys often irregular in shape or inclined to parallels and meridians. For each map area we sampled the gravity grid at each multibeam grid point and then projected these data with an Oblique Mercator projection centered on the multibeam survey and rotated to maximize the rectangular extent of the coverage. We interpolated the projected points onto regular grids with 1000 m spacing, and selected square subsets completely filled with data.

The cross-spectral coherence between the so-prepared multibeam and satellite gravity grids was computed with GMT (Wessel and Smith 1998) routine "gravfft" (J. Luis, personal communication, 2011), a generalization to the GMT routine "grdfft" which detrends the grids, tapers the edges, applies a two-dimensional Fast Fourier Transform, performs the coherence operation as detailed above, and outputs the coherence averaged azimuthally as a function of wavelength.

Results

The Australian-Antarctic Discordance (AAD) demonstrates our process. The AAD bathymetry (Figure 2a) has an axial trough and fine-scale abyssal hill topography offset by fracture zones, giving rise to gravity anomalies at both short and long wavelengths (Figure 2b). The cross-spectral coherence of these two grids is shown in Figure 3a. The results show coherence > 0.5 for wavelengths greater than 20 km. The coherence also appears to decrease at wavelengths greater than 100 km, a result expected due to isostatic compensation, though not reliably estimated because there are few coherence points at longer wavelengths.

In Figure 3b we show coherence results from all 25 regions. Some areas show low coherence at all wavelengths, while others are higher at longer wavelengths and taper to lower at shorter wavelengths.

Upward continuation of gravity from the sea floor to the sea surface attenuates anomalies with a wavelength λ by a factor $\exp(-2\pi d/\lambda)$, where d is the mean depth in the region. For most of the ocean, gravity from seafloor topography diminishes rapidly at wavelengths shorter than about 20 km. We therefore computed the mean coherence in the 20–160 km waveband to get a single value to characterize each area. The colors used in Figures 1, 3b, and 4 illustrate these mean coherences.

The mean coherence varies with geologic setting (Figure 1). Seamounts and slow-spreading ridges have high (> 0.7) mean coherence, other ridges and trenches have intermediate (0.5 - 0.7) coherence, and continental shelves have lower (< 0.5) values.

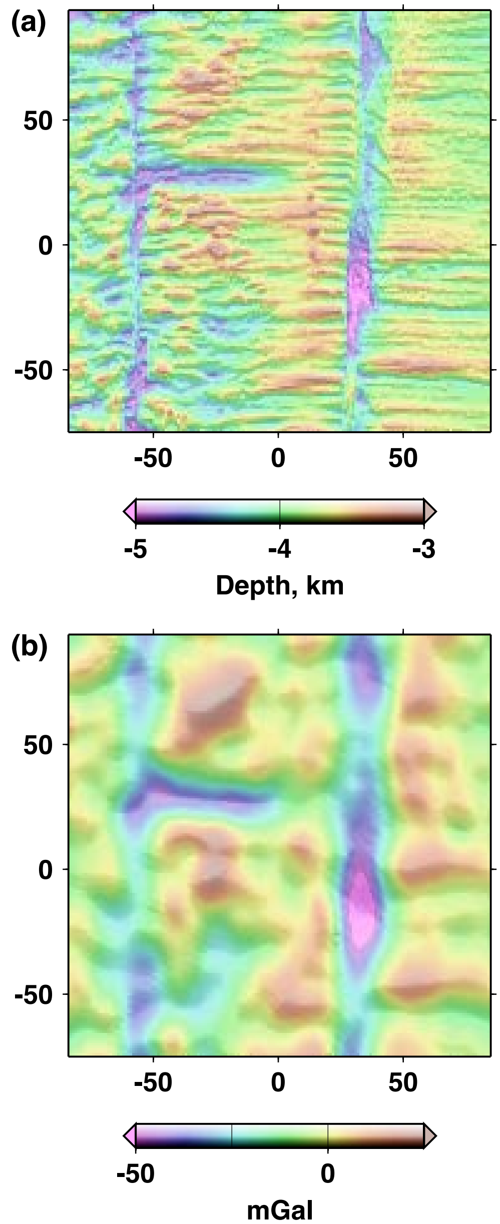

Fig. 2 Subset of grids of **a** regional multibeam bathymetry and **b** satellite-derived gravity over the Australian-Antarctic Discordance, for input to cross-spectral coherence analysis. AAD is notated in Fig. 1

Discussion and Conclusions

Smith and Sandwell (1994) devised a simple bathymetric estimation method that has two steps: (1) a spectral projection operator that transforms sea surface gravity anomalies into anomalies that may be tested for correlation with seafloor topography; and (2) a test for correlation, which yields a scaling factor that converts milliGals of projected gravity into meters of estimated topography. Our results are used to examine aspects of both steps.

The spectral projection operator is a linear and radially symmetric filter having two components: a downward continuation operator that amplifies anomalies with a wavelength λ by a factor $\exp(+2\pi d/\lambda)$, where d is again the mean depth in the region, and a band-pass filter, designed to select a range of length scales over which one may seek to correlate spectrally projected gravity with sea floor topography. Because short wavelengths grow exponentially with downward continuation, the short-wavelength cutoff of the filter is crucial. If the cutoff is too large, not enough gravity signal will be passed, underestimating and over-smoothing the estimated topography. If it is too short, exponentially amplified noise will obscure the true details of the estimated topography.

Because there were insufficient available data to determine the short-wavelength cutoff empirically, Smith and Sandwell (1994) shaped the band-pass filter using the signal-to-noise ratio in Geosat altimetry (Sandwell and McAdoo 1990), making it a simple

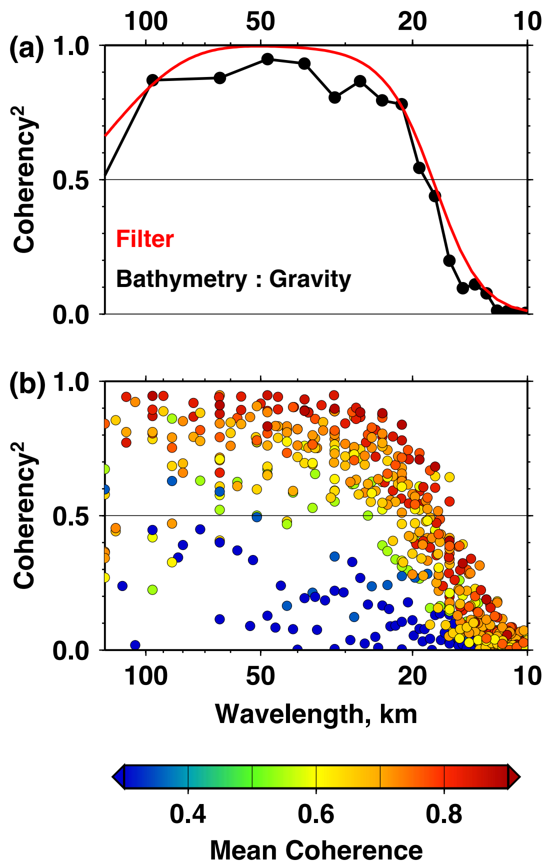

Fig. 3 **a** Azimuthally-averaged coherence (*black line*) between the grids shown in Figure 2. *Red curve* is the filter used by Smith and Sandwell (1994) to select gravity for bathymetric prediction. **b** Azimuthally-averaged coherences for all 25 regions. Each region's result is plotted with a constant color (shown in *color bar*), indicating the region's mean coherence

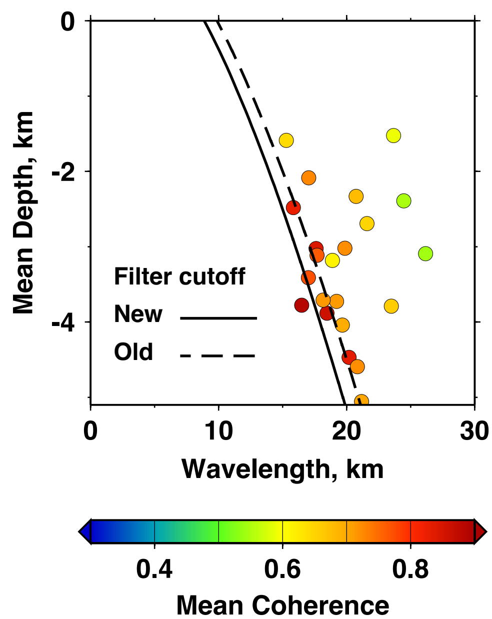

Fig. 4 Mean coherence (*colored circles*) plotted against mean multibeam survey depth and wavelength at which coherence crosses 0.5. *Black curves* indicate cutoff wavelength of Smith and Sandwell's (1994) bathymetric prediction filter, as it is now (*solid*) and was originally (*dashed*)

function of the local mean depth. The red curve in Figure 3a shows the filter shape, using the mean depth of the AAD, compared to our coherence results. The filter is correctly selecting coherent "signal" and rejecting "noise" in the AAD. The filter has a maximum value of 1, whereas our coherence maximum is somewhat less, suggesting that not all the gravity anomaly should be used to predict depth. This is dealt with in second step of their method, where the scale factor effectively reduces the amplitude of the prediction as needed.

To compare our results to their filter in all areas, we computed the wavelength at which our coherence estimates cross 0.5. Figure 4 shows that this wavelength decreases with decreasing mean depth and with increasing mean coherence. We also find this wavelength is < 20 km in areas of high mean coherence, 20 – 30 km in intermediate areas, and > 30 km or undefined in low areas (not shown).

The curved lines in Figure 4 show how the short-wavelength cutoff of the filter varies with mean depth. The dashed line follows the original filter of Smith and Sandwell (1994) based on the signal-to-noise ratio in altimetric gravity as it was then. The solid line is the filter they use today, with a revised value reflecting the improvements in altimeter signal-to-noise made by "retracking" the radar echoes (Sandwell and Smith 2009). The filter cutoff wavelength closely fits areas having the highest mean coherence, thus passing signal and rejecting noise in areas with high correlation between gravity and seafloor topography. We find that the prediction filter is doing a good job.

We recognize that the theoretical relationship between topography and related gravity is non-linear (Parker 1973), but in practice these anomalies lie outside our waveband of interest (Marks and Smith 2007).

We explain the observed variation of mean coherence with tectonic setting as follows. Seamounts, and rugged topography characteristic of slow spreading ridges, give rise to short-wavelength (<160 km) gravity anomalies that are highly correlated with underlying topography and hence produce high mean coherence. Medium and fast spreading ridges have relatively smooth topography (MacDonald 1982) and subdued gravity anomalies, and thus lower signal, accounting for their medium mean coherence. Small-scale features superposed on deep trenches may have signal attenuated by upward continuation and sediment cover, also giving medium mean coherence. Continental shelves are mostly flat, with gravity anomalies reflecting sub-surface density structures, so the mean coherence is low.

As satellite gravity fields are updated and improved, we will be able to repeat our coherence analyses and evaluate whether an even smaller filter cutoff wavelength is warranted, potentially leading to global bathymetric models resolving even finer-scale details of the seafloor.

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